Differentiation

Time : 1 hour

Total Marks : 35

(1) Differentiate with respect to x:

$$\frac{\sin x - x\cos x}{x\sin x + \cos x} \quad -(4)$$

(2) If $f(x) = \sqrt{x^2+1}$, $g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x-3$, then find

$f'(h'(g'(x)))$. $-(4)$

(3) Differentiate

$$\cot^{-1}\left(\frac{1-x}{1+x}\right) \quad -(4)$$

(4) Find $\frac{dy}{dx}$: $(x^2+y^2)^2 = xy$ $-(4)$

(5) If $e^x + e^y = e^{x+y}$

Prove that $\frac{dy}{dx} = -\frac{e^x(e^y-1)}{e^y(e^x-1)}$ or, $\frac{dy}{dx} + e^{y-x} = 0$ $-(4)$

(6) If $x^m y^n = (x+y)^{m+n}$, Prove that $\frac{dy}{dx} = \frac{y}{x}$ $-(5)$

(7) If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$

Show that at $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{b}{a}$ $-(5)$

(8) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $-1 < x < 1$, $x \neq 0$

$-(5)$